

# **METHODOLOGY FOR CONVEYANCE ESTIMATION IN TWO-STAGE STRAIGHT, SKEWED AND MEANDERING CHANNELS**

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Reliable estimates of discharge capacity are essential for the design, operation and maintenance of open channels, and more importantly, the prediction of flood levels. The conveyance estimation methods that are employed in commercially available river modelling software are principally based on historic hand-calculation formulae, with little or no account taken of the more recent advances in knowledge and understanding. A scoping study, commissioned by the British Environment Agency, identified the need to reduce the uncertainty associated with flood level prediction through incorporating this recent research into a Conveyance Estimation System (called the "CES"). The development of the CES is now underway, involving a partnership between academic researchers, experts and users. This paper describes the calculation methodology that has been adopted, which is applicable to all river and floodplain morphologies, considers all the physical flow processes that are present and where necessary, includes empirical or calibration coefficients that are based on previous research and expert advice. The approach uses a depth-integration of the Reynolds Averaged Navier-Stokes equations for flow in the streamwise direction, and generates a numerical solution using the finite element method. Emphasis is given to the secondary flow energy loss mechanisms and their effect on the lateral velocity distribution. The depth-averaged velocity predictions are compared with measured data from the Flood Channel Facility at HR Wallingford, the University of Bristol 1:5 scale model of the River Blackwater and the River Seven at Shrewsbury.

**Keywords:** Conveyance, depth-averaged velocity, secondary flow losses, meandering channels

## **INTRODUCTION**

Flood level predictions are fundamental for flood warning, determining the development approach to be adopted in flood-risk areas and the long-term management of rivers. High flows may result in increased sediment transport rates and local scour, thus influencing the hydraulic design of structures such as bridge piers and abutments, while low flow conditions are necessary for channel preservation such as weed cutting, dredging and removal of blockages.

Conveyance estimation methods currently employed in river modelling software are based on historic hand calculation formulae such as Manning's equation, which was published in 1889. More recent work, including the substantial research programme on the EPSRC Flood Channel Facility (FCF) at HR Wallingford, has provided significant improvements in the understanding and calculation of channel conveyance. This ranges from the understanding and interpretation of the complex flow mechanisms, to the advent of computing tools that enable more sophisticated solution techniques. An extensive database comprising both physical model and real river measurements provides a sound basis for testing these research advances. Significant

contributions include those of Chang (1983), Ervine & Ellis (1987), Shiono & Knight (1989), Ackers (1991, 1993), James & Wark (1992), Bousmar & Zech (1999) and Ervine *et al* (2000).

In 2000, a scoping study entitled “*Reducing Uncertainty in River Flood Conveyance*” was commissioned by the Environment Agency, which identified the need to reduce the uncertainty associated with flood level prediction through incorporating these recent advances in knowledge. The key outcome was a Targeted Programme of research for the development of a Conveyance Estimation System (CES). The CES will comprise three core components: (i) the Conveyance Generator (determines stage-discharge relationship) (ii) the Roughness Advisor (provides local vegetation/substrate roughness) and (iii) the Uncertainty Estimator (provides some measure of the uncertainty associated with the predicted water level). This paper describes the calculation methodology that has been adopted for the Conveyance Generator (CG) component, and in particular, the proposed model for the secondary flow energy loss terms.

### CALCULATION APPROACH

The calculation approach is based on the depth-integrated Reynolds Averaged Navier-Stokes equations for flow in the streamwise direction. It extends the original Shiono-Knight Method SKM (Shiono & Knight, 1989) for straight prismatic channels, to include the more recent approach of Ervine *et al* (2000) for meandering channels. The depth-averaged form of the Navier-Stokes equation for a small element within the cross-section of an open channel, with a bed inclined in the streamwise direction (Shiono & Knight, 1989), is

$$\rho gHS_o - \frac{\rho \beta f U_d^2}{8} + \frac{\partial}{\partial y} \left\{ \rho \lambda H^2 \left( \frac{f}{8} \right)^{1/2} U_d \frac{\partial U_d}{\partial y} \right\} = \frac{\partial}{\partial y} [H(\rho \overline{UV})_d] \quad (1)$$

where  $(U, V)$  are velocity components in the  $(x, y)$  direction,  $x$  is streamwise parallel to the bed and  $y$  is lateral across the channel,  $H$  is the local depth measured normal to the channel bed,  $\rho$  is the fluid density,  $g$  is the gravitational acceleration,  $S_o$  is the reach averaged longitudinal bedslope,  $\beta$  is a coefficient to account for the influence of local bedslope on the bed shear stress,  $\lambda$  is the dimensionless eddy viscosity,  $f$  is the friction factor and the suffix ‘ $d$ ’ indicates a depth-averaged value. For solution of the depth-averaged velocity  $U_d$ , the difficulty arises as  $(\overline{UV})_d$ , in the secondary flow term on the right hand side of equation (1), is unknown. Thus strategies for providing a “closure” for this term from the primary flow variables is needed. The SKM approach of Shiono & Knight (1989) defines a secondary flow term  $\Gamma$ , where

$$\Gamma = \rho (\overline{UV})_d \quad (2)$$

which is a calibration parameter that varies with  $y$  and relative depth. Ervine *et al* (2000) propose an alternative model for the more complex secondary flows in meandering channels. The secondary currents are related to the depth mean velocity by a coefficient  $C_{uv}$  such that,

$$\overline{UV} = C_{uv} U_d^2 \quad (3)$$

which is applied within the main channel only. This model introduces a non-symmetrical effect on the lateral depth-averaged velocity distribution. The conveyance methodology presented here seeks to combine these two approaches such that for straight, transitional and fully meandering channels, the secondary flow term is composed of  $\Gamma$ , a linear combination  $\Gamma$  and  $C_{uv}$ , and the  $C_{uv}$  model respectively (Figure 1).

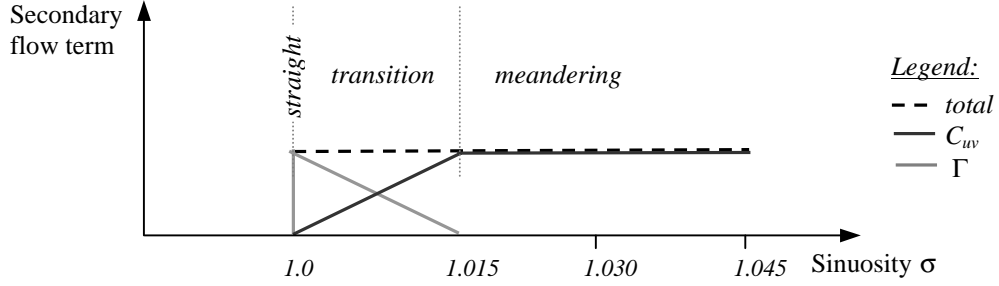


Figure 1: Contributions from the secondary flow terms with increasing sinuosity

Previous research (Samuels, 1989) advocated solution for the unit flow rate  $q (=U_dH)$  as the primary variable, instead of the depth-averaged velocity  $U_d$ , due to the strong continuity properties of  $q$  at abrupt changes in channel depth e.g. a vertical step in an engineered channel. Expressing equation (1) in terms of the unit flow rate, and introducing the relationship for the secondary flow terms depicted in Figure 1, yields

$$\underbrace{gHS_o}_{(I)} - \underbrace{\frac{f\beta q^2}{8H^2}}_{(II)} + \underbrace{\frac{\partial}{\partial y} \left[ \lambda H \left( \frac{f}{8} \right)^{\frac{1}{2}} q \frac{\partial}{\partial y} \left( \frac{q}{H} \right) \right]}_{(III)} = \underbrace{\frac{(1.015 - \sigma)}{0.015} \Gamma + \frac{(\sigma - 1.0)}{0.015} C_{uv}}_{(IV)} \frac{\partial}{\partial y} \left[ \frac{q^2}{H} \right] \quad 1.0 \leq \sigma \leq 1.015 \quad (4a)$$

$$gHS_o - \frac{f\beta q^2}{8H^2} + \frac{\partial}{\partial y} \left[ \lambda H \left( \frac{f}{8} \right)^{\frac{1}{2}} q \frac{\partial}{\partial y} \left( \frac{q}{H} \right) \right] = C_{uv} \frac{\partial}{\partial y} \left[ \frac{q^2}{H} \right] \quad \sigma > 1.015 \quad (4b)$$

- (I) variation in hydrostatic pressure along reach
- (II) boundary friction effects
- (III) turbulence due to shearing between vertical layers
- (IV) turbulence due to secondary currents (straight and meandering channels)

where the sinuosity  $\sigma$  is defined as the thalweg length over the valley length. The bedslope coefficient  $\beta$  is evaluated from,

$$\beta = (1 + S_y^2)^{\frac{1}{2}} \quad (5)$$

where  $S_y$  is the lateral bedslope. Calibration of this model requires the parameterisation of the coefficients  $f$ ,  $\lambda$ ,  $\Gamma$  and  $C_{uv}$ . The distribution of  $f$  is based on a form of the Colebrook-White equation, with coefficients applicable to open channels. The absolute roughness value,  $k_s$ , is

based on the local friction due to substrate, vegetation and/or artificial cover, as the energy losses due to shearing between the vertical layers, secondary flows and planform geometry are incorporated elsewhere. The distribution of the dimensionless eddy viscosity  $\lambda$  is given by (Abril, 2002),

$$\lambda = \lambda_{mc} (-0.2 + 1.2 D_r^{-1.44}) \quad (6)$$

where the main channel value,  $\lambda_{mc}$ , is 0.24 and the relative depth  $D_r$  is defined as the local depth  $H$  over the maximum depth. The inbank (i) and overbank (o) distributions of  $\Gamma$  are given as (Abril, 2002),

$$\Gamma_{mc(o)} = 0.15 H \rho g S_o \quad \Gamma_{fp(o)} = -0.25 H \rho g S_o \quad \Gamma_{mc(i)} = 0.05 H \rho g S_o \quad (7a,b,c)$$

however, the transition from inbank to overbank flow conditions is not clearly defined. The  $C_{uv}$  parameter has previously been defined as a function of relative depth, relative roughness and sinuosity (Ervine *et al*, 2000), however no robust relationship exists. Expansion of the  $C_{uv}$  term gives:

$$\rho C_{uv} \frac{\partial}{\partial y} (H U_d^2) = \rho C_{uv} H \frac{\partial}{\partial y} (U_d^2) + \rho 2 C_{uv} U_d \frac{\partial H}{\partial y} \quad (8)$$

The first term on the right hand side introduces a damping effect to the depth-averaged velocity profile i.e. it smoothes or flattens out the profile. The second term introduces a non-symmetrical velocity profile through the lateral gradient of the local depth, such that this term is small on falling slopes (concave down effect), zero for horizontal channel beds (the flattening of the first term is present) and large on rising slopes (concave up effect).

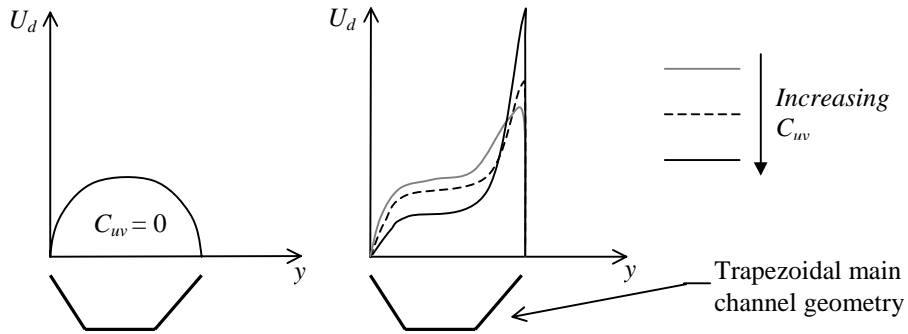


Figure 2: Effect of the secondary flow term  $C_{uv}$  on the depth-averaged velocity profile

The magnitude and sign of  $C_{uv}$  influences the size and orientation of the resulting skew respectively. Figure 2 illustrates the effect of increasing positive  $C_{uv}$  values on the depth-averaged velocity profile, for a trapezoidal channel geometry. On the laterally rising channel side slope, the  $dH/dy$  term causes an unrealistic increase and spike in the depth-averaged velocity. However, the falling side slope and main channel ‘skew’ effect is similar to that observed in real meandering channel data. The Conveyance Generator has thus introduced the  $C_{uv}$  model for

lateral slopes that are less than or equal to zero. Figure 3 illustrates the final distributions of the calibration parameters for a trapezoidal two-stage channel.

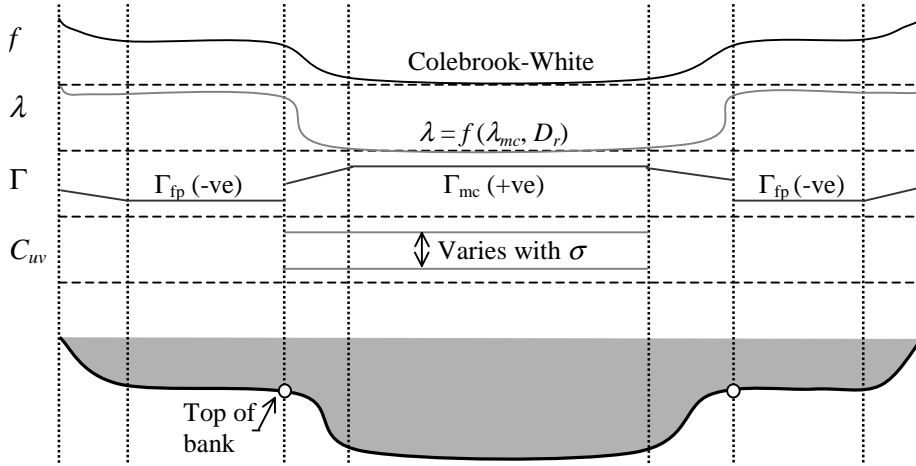


Figure 3: Lateral distribution of calibration parameters within the Conveyance Generator

### SOLUTION TECHNIQUE

Equation (4) is a non-linear, elliptic, second order partial differential equation, which is solved numerically with the finite element method, which is well suited to the solution of elliptic equations. The cross-sectional area of flow represents the solution domain, which is discretised laterally into a number of elements, and the variable  $q$  is replaced with piecewise linear approximations. The solution to resulting system of discrete equations is generated through an iterative procedure, linearised in the correction term  $\Delta q^n$ , to update unit flow rate  $q^{n+1}$  from the known  $q^n$  value from the previous iteration. The Dirichlet boundary condition,  $q = 0$ , is prescribed at the boundary nodes at the edges of the flow domain. The iteration procedure is designed to converge nearly quadratically.

### RESULTS OF EXPERIMENTAL AND FIELD DATA TESTS

The Conveyance Generator methodology was tested against small and large-scale flume data, as well as real river measurements purpose-made for research. The velocity predictions are compared to data from the FCF Phase A & B experiments, the River Blackwater 1:5 scale model (Sellin *et al*, 1992) and the River Severn downstream of Shrewsbury (Ervin DA, personal communication). The flow parameters are summarised in Table 1. Column 2 provides both the measured and predicted flow rates, of which the maximum difference is 2.4%.

Data Set	Discharge	Flow depth (m)	Slope $\times 10^{-3}$	Roughness $k_s$ (mc, fp)	Sinuosity	$C_{uv}$	% $\Gamma$ term	% $C_{uv}$ term
	Q (m <sup>3</sup> /s) Actual / CG							
FCF Series A14	0.261 / 0.260	0.176	1.027	0.00005	1.0038	1.0	50	50
FCF Series B24	0.634 / 0.627	0.250	0.996	0.00070	1.3740	3.0	0	100
River Blackwater 1:5 model	0.124 / 0.121	0.187	0.847	0.01400	1.1800	2.5	0	100
River Severn (Shrewsbury)	103.0 / 104.7	7.620	0.146	0.3, 0.18	1.0800	0.8	0	100

Table 1: Summary of two-stage data sets for testing of the Conveyance Generator

The depth-averaged velocity predictions are shown in Figure 4 (a-d), where the cross-section in (a) is situated in a channel with the main channel skewed at 5 degrees to the floodplain, and the cross-sections in (b-d) are taken at the bend apexes.

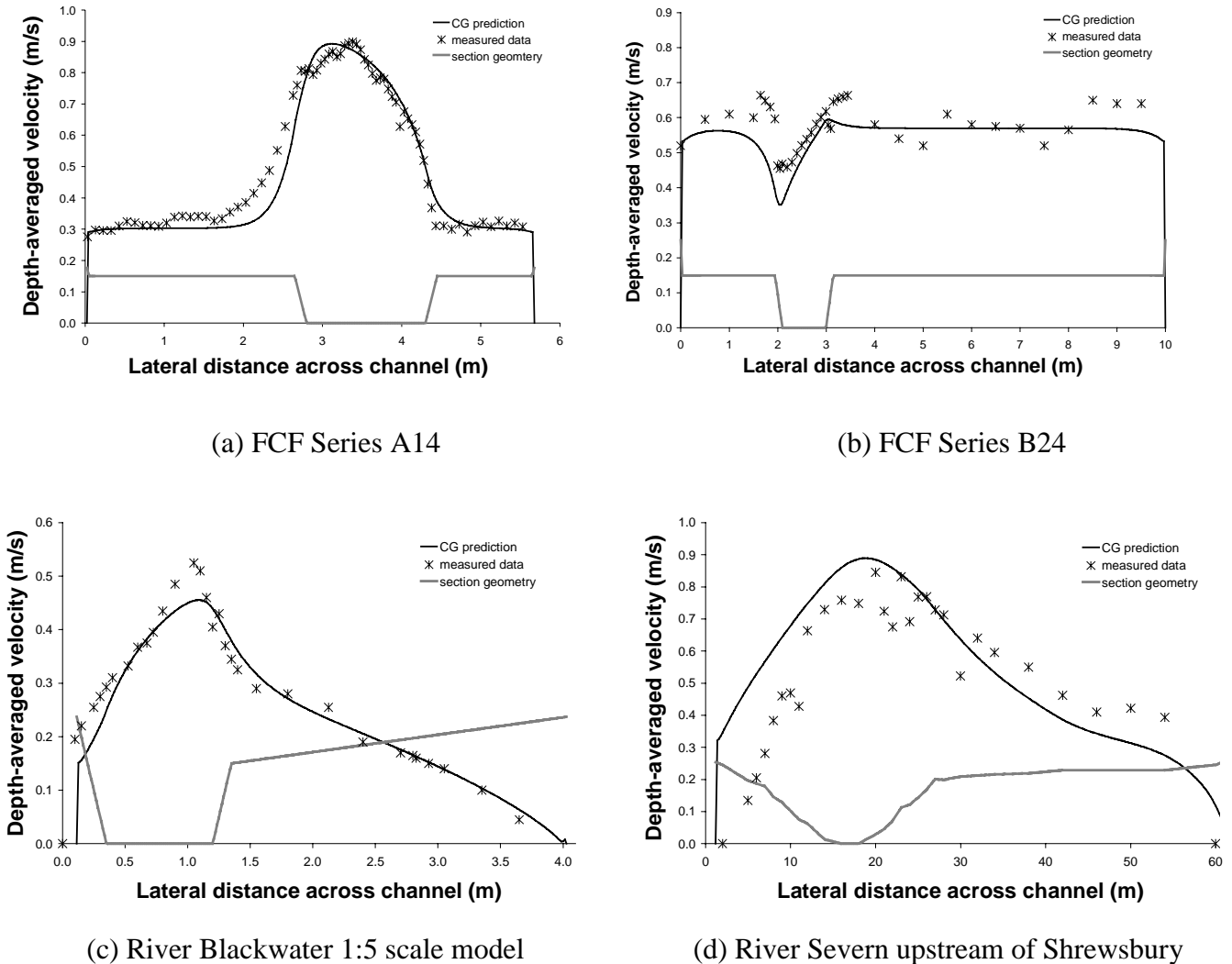


Figure 4: Comparison of the lateral depth-averaged velocity CG predictions to observed data

Figure 5(a) shows the regression analysis of the %  $C_{uv}$  and the sinuosity, for overbank flow, including additional data from the University of Glasgow (Ervin DA, personal communication) and the River Blackwater in Hampshire (Sellin RHJ, personal communication). The regression line has an  $R^2 \approx 0.98$ , suggesting a reasonable fit to the calibrated data. A  $C_{uv}$  value of approximately 1% for  $\sigma = 1.015$  can thus be extrapolated. Figure 5(b) compares the hypothesis of the linear combination of the two secondary flow terms, with a  $C_{uv}$  value of 1% at  $\sigma = 1.015$ , to the calibrated values. Two of the data points are scattered such that the gradient of the proposed model appears too steep. It may therefore be preferable for channels within this small sinuosity range to use a fixed combination of 0.6 times  $C_{uv} = 1\%$  and 0.4 times the  $\Gamma$  term. However, for continuity, as straight channels have a sinuosity of 1.0 (the third data point), the linear

combination is preferable. A further alternative is to fit a curve to the calibrated data. Given the small range of sinuosity values in the transitional region, together with the uncertainty of accurately estimating the sinuosity within this small range, the linear transition is considered adequate.

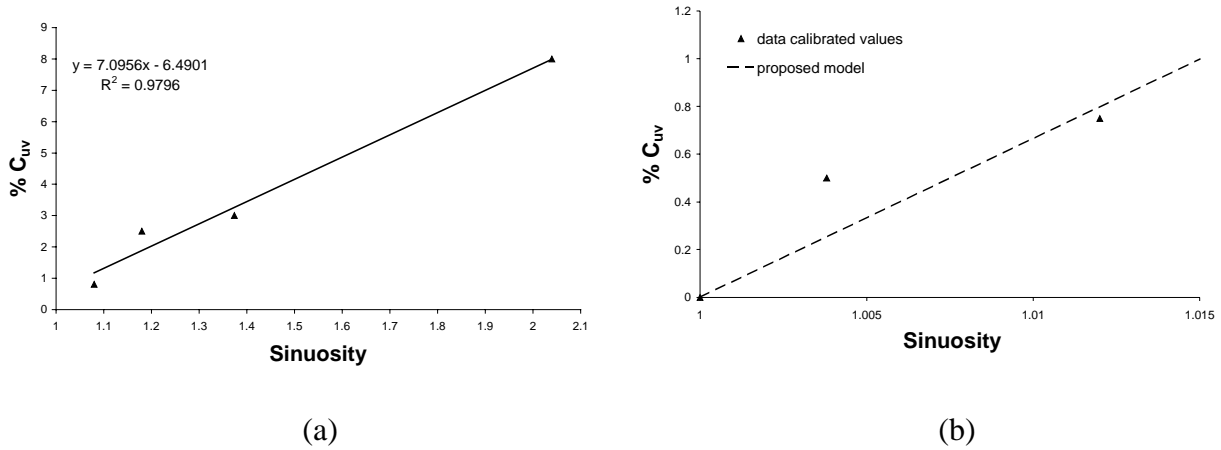


Figure 5: (a) Regression analysis of  $C_{uv}$  for sinuosities greater than 1.015 and (b) comparison of the proposed model for  $1.0 < \sigma < 1.015$  to the calibrated values

## CONCLUSIONS

The calculation methodology for the Conveyance Generator has been described in detail. Three of the four calibration parameters,  $f$ ,  $\lambda$  and  $\Gamma$  are well defined for straight prismatic channels from previous research. A model had been suggested for the combination of the two secondary flow parameters,  $\Gamma$  and  $C_{uv}$ , for two-stage meandering channels. The data suggests the model is reasonable for sinuosities in the small range 1.0 to 1.015. The regression analysis of the calibrated  $C_{uv}$  values for sinuosities greater than 1.015 provides a reasonable fit with an  $R^2$  value of 0.98. The predicted depth-averaged velocity profiles for channels of varying sinuosities correspond well with the non-symmetrical trend of the observed values for both experimental and real river data. The methodology outlined here is physically based, considers all the energy loss mechanisms, can be extended to skewed and meandering channels and has the added advantage of providing the depth-averaged velocity distribution. The Conveyance Generator will thus provide a substantial improvement on existing river modelling approaches to conveyance estimation.

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- Further details on the Conveyance Project (and related reports) are available on the project website: <http://www.river-conveyance.net/>.